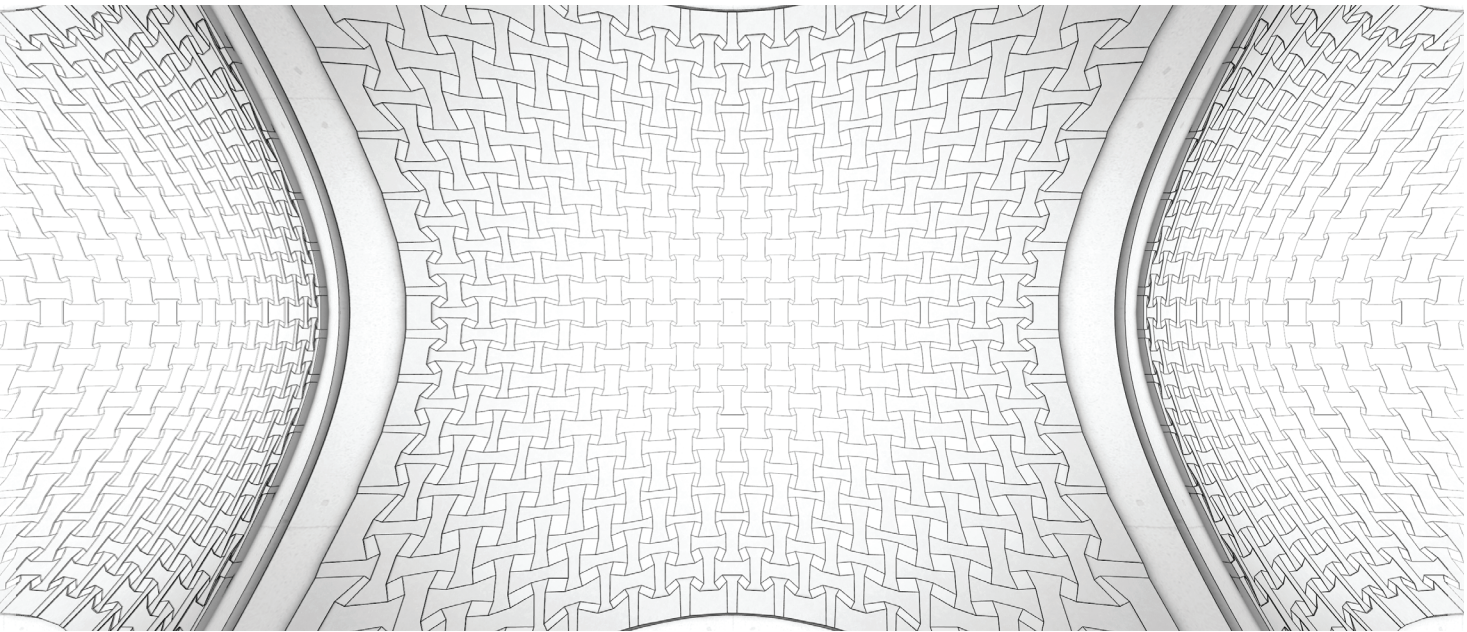


Automating Bi-Stable Auxetic Patterns for Polyhedral Surface

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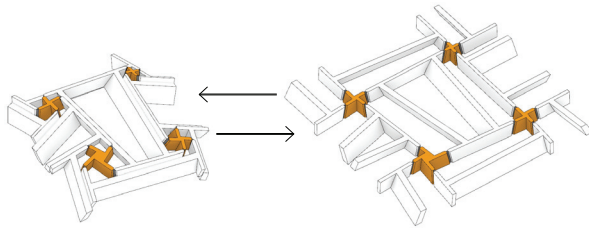
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ABSTRACT

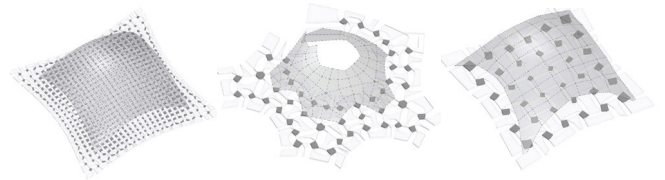
Bi-stable auxetic structures, a novel class of architected material systems that can transform bi-axially between two stable states, offer unique research interest for designing a deployable stable structural system. The switching behavior we discuss here relies on rotations around skewed hinges at vertex rotating connectors. Different arrangements of skewing hinges lead to different local curvatures.

This paper proposes a computational approach to design the self-interlocking pattern of a bi-stable auxetic system that can be switched between flat and desired curved states. We build an algorithm which takes a target synclastic polyhedral surface as input to generate the geometrical pattern with skewing hinges. Finally, we materialized prototypes to validate our proposed structures and to exhibit potential applications.

0 Perspective ceiling view of a proposed architectural dome surface realized by bi-stable auxetic patterns



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INTRODUCTION

Deployable structures are widely applied across many different disciplines, such as antenna design or emergency shelter construction (Häuplik-Meusburger 2011; Shah et al. 2019). From the architectural perspective, increasing demand for geometrically complex architecture pushes innovations of efficient building formworks to save fabrication time, labor, and cost. Deployable structures, which can transform from a flat state to the desired target geometry, are plausible solutions and have thus received intensive research interest for many decades.

Although many designs of deployable structures have focused on regular patterns and repetitive mechanical joints in the past, recent work has leveraged geometric knowledge and advanced manufacturing technology to produce more sophisticated systems to create free-form target geometry. By compositing material with different mechanical properties (e.g., stiffness, expansion ratio) or designing mechanical linkages, structural systems can transform from the original fabricated state to a target configuration state by simply applying an external trigger.

This investigation is inspired particularly by two earlier studies: Rafsanjani and Pasini (2016) presented a family of bi-stable auxetic mechanisms that have homogeneous expansion rates; Ou et al. (2018) identified how to design spatial transformation unit based on skewed rotational axes.

Contributions—We introduce a fully automated algorithm to generate parameterized bi-stable auxetic patterns for target synclastic geometry. We investigate a hinge-based deployable structure systems to achieve bi-stable shape reconfigurations from flat to curved state. We achieve the flat-to-curved transformation by skewing hinge directions (Figure 1). The resulting pattern in flat configuration includes cutting paths along 2D curves with various inclinations. We further show the geometric constraints brought by the bi-stable and auxetic mechanisms, especially within the design of a vertex star, where one skewing hinge directly constrains the design of its neighborhood hinge

- 1 A 3D-printed prototype for proposed system which combines flat pattern from Rafsanjani and Pasini 2016 and spatial rotational axes from Ou et al. 2018
- 2 Three different input target geometry surfaces and their generated bi-stable auxetic patterns

along a corresponding edge. Our computational method is tested and validated through various input surfaces (Figure 2).

RELATED WORKS

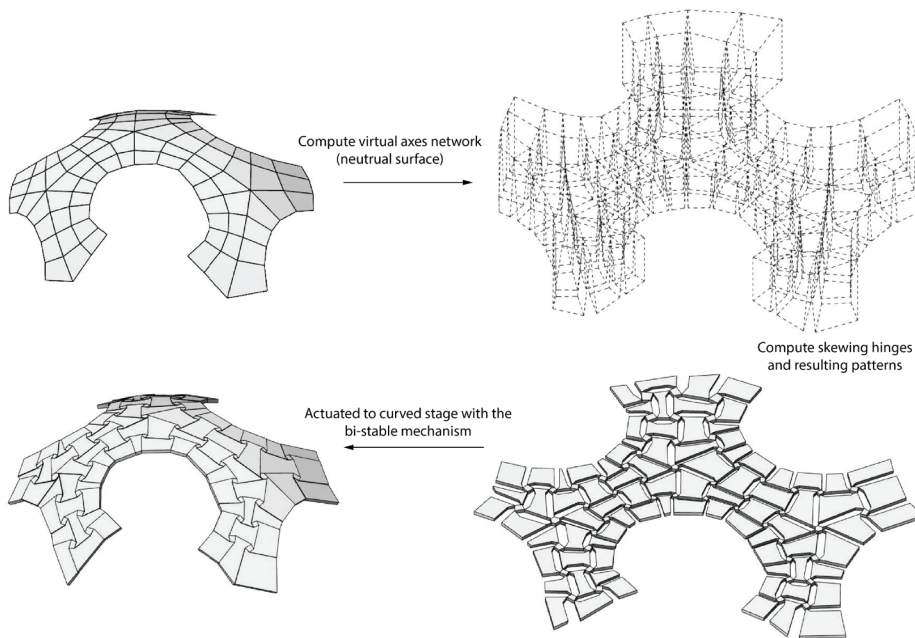
We survey previous research projects featuring deployable systems in this section with a special focus on work exhibiting external hinge systems as well as the latest research progress on bi-stable auxetic systems.

Kinetic-based Reconfiguration

Researchers have focused on the kinetic behavior of mechanical systems. They applied linear and spherical hinge joints, which respectively allow 1 and 3 degrees of freedom (DoFs). Origami is perhaps the most famous reconfigurable system involving linear hinges (Tachi 2011). For the making of more free-form surfaces, cuts or slits can be introduced to the origami folding system. The result is called kirigami. Researchers have demonstrated its ability to make any free-form surfaces by programming the patterns properly (Liu et al. 2018, 2019; Jiang 2020). Rotational ball hinges can provide more DoFs in transformation, making it easier to achieve target free-form geometry. For instance, Konaković et al. (2016) presented how to design a free-form surface using the triangular auxetic pattern from conformal mapping and geometry optimization.

Bi-stable Mechanism

Mechanical bi-stability is a system that is also termed as snap-through buckling (Vahidi and Huang 1969). The term reflects on the features of how the system is switched from one stable state to another. From an energy point of view,



3 General workflow of designing bi-stable auxetic patterns: We first take a polyhedral surface with synclastic curvature as input, then compute the virtual axes network that each face will rotate against. The rotary network satisfies the condition of 'neutral surface' to the input geometry so that rotation at each edge will have bi-stable properties. Subsequently, we compute skewing hinges at each rotary vertex to generate resulting patterns. Finally, the geometry with such patterns will push back to curved state given external forces.

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as illustrated in Figure 4 with the basic bi-stable unit, the stable states correspond to the local minimum points of the system's energy-displacement graph. When an external force applies to the system, two members begin to deflect. Until the external load passes the critical point, the unit will suddenly deviate from critical state to the alternative state. After the force is removed, elasticity will bring material back to its rest length, resulting in mirrored geometry to its starting state. To make full use of this geometry feature, the hinges should ideally store no energy in the rest state. In this paper, all the hinges are assumed to behave ideally, in order to design bi-stable mechanisms with geometric principals.

Bi-stable Auxetic System

Recent research has considered auxetic behavior in the bi-stable system, resulting in useful functions to explore a wider range of reconfiguration system. Rafsanjani and Pasini (2016) have shown a surface with periodic bi-stable units that can also be auxetic with a Poisson's ratio of -1 . Chen et al. (2021) used parametric cells with different expansion ratios to achieve the curved bi-stable auxetic surface at a fixed boundary. The primary difference in our research is that all the panels can spring back to their original shapes without significant residual strain. We put our primary focus on the geometry of the start and end configurations. Our work guides the deployed state towards a designed shape by adequately arranging the rotation axes. We see our work as a novel demonstration of an inverse algorithm for designing the bi-stable deployable surface system.

METHOD OVERVIEW

This section introduces the fundamental approach and basic design workflow of our deployable system. The whole transformation process can be divided into two hierarchical perspectives: 1) in local reference frames, where two adjacent panels rotate against their corresponding virtual axis to achieve a target dihedral angle; 2) in the world reference frame, where a network created by connecting all the virtual axes goes through an isometric transformation, such as with a mirror relationship to the original stable stage. Combining the two transformations together will deliver a collection of flat faces to a curved configuration. Reversely, an inverse transformation dispatch faces from a curved mesh to discrete pieces in a plane.

The critical principle of deployment in our approach is to program dihedral angles between adjacent panels through out-of-plane rotations from rotating connectors around each vertex. After we generate the layout of different faces in the plane, we can program the hinge locations and directions at each rotator according to the corresponding dihedral angle and the positional constraints from neighboring rotators. This algorithm will be further explained in the *Computational Workflow* section.

Figure 3 illustrates the general workflow of our inverse design method. In the first step, the facets of the polyhedral mesh are distributed onto a plane. Each face can rotate around an axis in a network which lies outside the plane. In a mesh where Gaussian curvature at each vertex is positive, the network exists and is called a 'neutral surface'

(Chiang 2019). In the second step, we map the hinge locations and directions based on the layout faces in a plane. A group of hinges around one vertex forms a rotating connector. If the vertex has positive Gaussian curvature, the rotating connector is a truncated pyramid; if it has negative Gaussian curvature, the rotating connector is a truncated tetrahedron. We calculate the shape of the rotating connector on each vertex and propagate the result from a central vertex to everywhere in the map. These rotating connectors naturally define the shape of panels, by defining either full hexagonal gaps or seams or half hexagonal seams (Figure 5). In the final step, an external pulling or pushing force actuates all the panels to rotate around their rotating connectors towards their second stable stage.

GEOMETRIC DEMONSTRATION

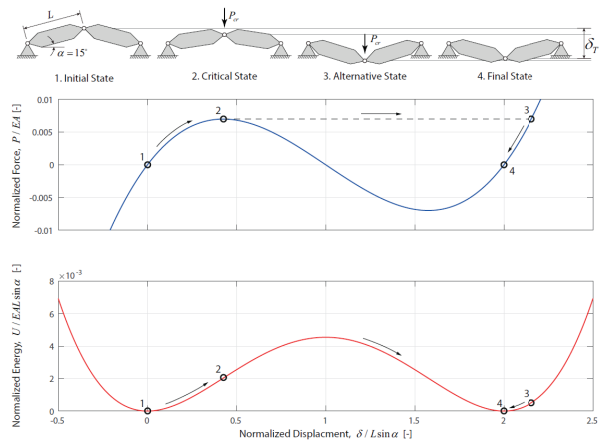
Kinetics of Bi-stable Auxetic Pattern

We analyze the geometry and kinetics behavior in auxetic bi-stable tiling first proposed in Rafsanjani and Pasini (2016) in Figure 6. The auxetic deformation behavior emerges due to the rotation of the colored squares; thus, we call them 'rotating connectors', and we call the rest of the components 'panels'. The proposed patterns create a hexagonal void for each edge. Two ends of the hexagonal void are two "anchor" points that define the mirror axis of the void. This mirrored relationship echoes the nature of bi-stability from a geometric perspective in our previous section.

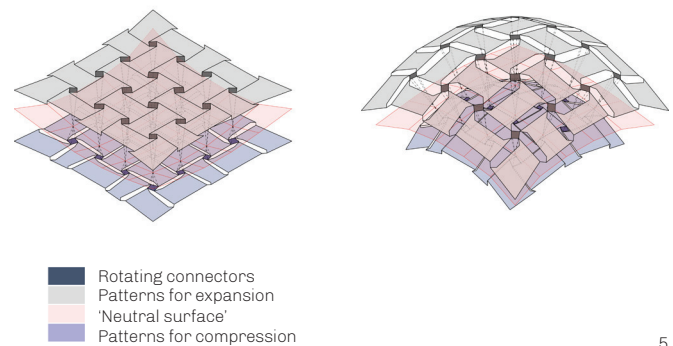
In planar transformation, like the cases in Rafsanjani and Pasini (2016), all the rotation axes are along the normals of the planes. Jifei et al. (2018) propose that skewing the rotation axis in space produces out-of-plane rotation. Inspired by this, we replaced the homogeneously repeated pattern and the perpendicular cuts with a heterogeneously graded pattern and tilted cuts for flat to curved reconfiguration. By this method, two adjacent panels will form a dihedral angle when the void opens or closes, corresponding to the expansion or contraction stage in the auxetic transformation. The following sections will discuss the geometric behavior of this mechanism.

Edge Transformation: Hexagonal Void with Virtual Hinge

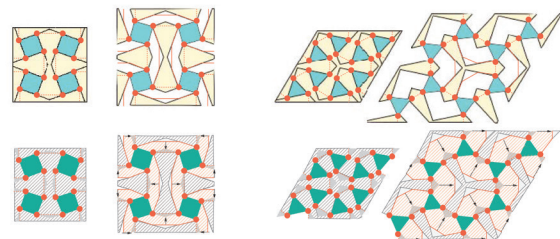
We refer to the research by Chiang et al. (2018), which achieved a target dihedral angle by a bi-stable mechanism system through a configuration of skewed hinges. The author demonstrated important properties of spatial bi-stable transformation within a unit of two adjacent panels: the generalization of the prismatic voids in Rafsanjani and Pasini (2016) into non-prismatic voids. The generalized voids still have heptagonal basis



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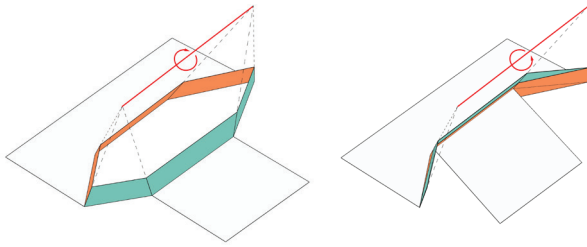
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- 4 The reconfiguration process of the idealized bi-stable mechanisms and their load-displacement response from Chiang (Chiang 2019)
- 5 By defining hexagonal gaps (bottom part), or half hexagonal seams (top part), the bi-stable auxetic pattern can be actuated through expansion or compression
- 6 Bi-stable unit in plane contains a set of 'rotating connectors' (green) and a set of 'panels' (yellow), initially studied by Rafsanjani and Pasini (2016)

and are symmetric, but are capable of delivering spatial (i.e. non-translational) transformations (Figure 7). The connecting panels have different rotating arms at the top and bottom surfaces, leading to different displacements at the two surfaces. The length of the rotating arms is proportional to the distances from the rotation axis, so is the displacement.



7 Edge transformation is illustrated as closing the gap of a hexagonal void; the closing/ opening reconfiguration is equivalent to rotate against a virtual hinge

8 Vertex transformation is illustrated as rotating around a polyhedron connector: (left/right) open(flat)/closed(curved) stage with a connector at an anticlastic vertex

9 (Left/right) Open(curved) /closed(flat) stage with a connector at a synclastic vertex

To inversely design the hinges of two given adjacent panels (i.e., the dihedral angle is given), we might need to encounter the displacement by extending, trimming, or offsetting the face of some panels. If the two faces stay at the original design position, the virtual hinge lies precisely on the bisecting plane of this dihedral angle to make sure that the flattened result will stay in the same plane. When one of the faces is offset along the normal, the virtual hinge no longer lies in the bisecting plane of two faces.

Vertex Transformation: Frustum Connector

When we reassemble the hexagonal voids along each edge, there are rotating connectors along vertices. In the case

by Rafsanjani and Pasini (2016), all the rotating connectors were right prisms since all the hinges are parallel and orthogonal to the plane. In the case of Figure 7, a virtual hinge exists in each hexagonal voids between two faces. Therefore, the connectors are frustums (i.e., truncated pyramids), and the local convexity defines their shapes.

7 Figures 8 and 9 exhibit different types of rotating connectors at the vertices. Each pair of hinges in the connector will intersect at a point either above or below the frustum, depending on the dihedral angle that its corresponding hexagonal void maps to. When the discrete Gaussian curvature at one vertex is positive, its connector is a frustum, as all hinges around meet at an apex. The scenario is more complex when the dihedral angle around one vertex has different signs, leading to negative Gaussian curvature. In the case that the vertex is at a valence of 4, its connector is a truncated tetrahedron.

Constraints in a Vertex Star: Simultaneous Rotations

The designers must consider the constraints between interrelated hexagonal voids and the connector to arrange the hexagonal voids around a vertex. Each hinge affiliates to two successive hexagonal voids from the relationship in two configurations. Consider the design of a vertex star in Figure 10. Observing the angles around one hinge point in open and closed states gives us two equations:

$$\alpha_n + \beta_n + \theta_n + \omega_{n-1} = 2\pi, \quad (1)$$

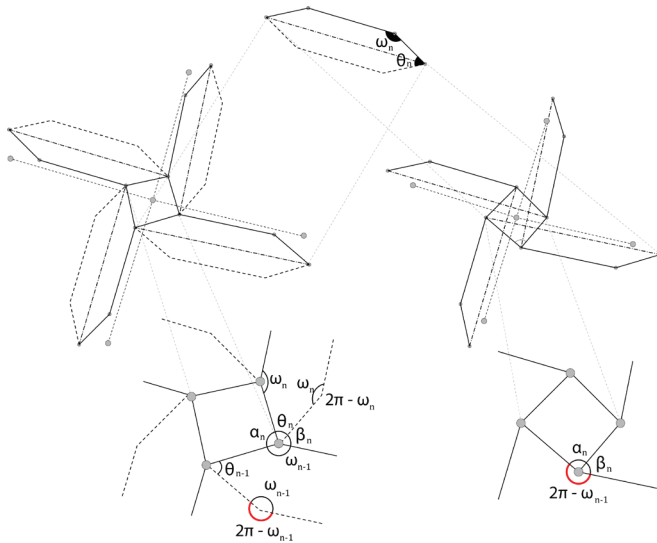
$$\alpha_n + \beta_n + (2\pi - \omega_{n-1}) = 2\pi, \quad (2)$$

where α is the interior angle of the rotating connector, β is the angle of the panel attaching to the connector, while ω and θ are the obtuse and acute angle of the hexagonal void, respectively. Subtract equation (2) from equation (1), we deduct the following equation which describes the angle agreement between the two successive voids:

$$\theta_n + 2\omega_{n-1} = 2\pi, \quad (3)$$

Equation (3) connects the degrees of freedom around a vertex. It means the geometry of a hinge is affected by the adjacent ones. In the meantime, the coplanar condition and dihedral angle correspondence given by the Edge Transformation subsection also limit the position of the hinge by the adjacent hinge in its neighborhood vertex.

To make our system more manageable, we introduce the method of unrolling a conical mesh (Liu et. al. 2006; Chiang et. al. 2018) as a guide for those rotating polygons. The conical mesh has nice properties as each vertex has a



10 Internal angle constraints for hinge positions within a vertex star

normal axis intersected by all the bisector planes of the dihedral angles between surrounding facets, which can be taken as an offset direction for generating our virtual hinges. We will set up the computational workflow based on the conical surface input.

COMPUTATIONAL WORKFLOW

The previous section analyzed the mechanism and geometric features of bi-stable auxetic reconfiguration. Here, we discuss how to automate the design of each hexagonal void and rotating connector. This paper focuses solely on the situation where each virtual rotation axis stays on the same side of the input polyhedral mesh (i.e., synclastic surface). Under such circumstances, the Gaussian curvature is positive, and all hinges around a rotating connector meet at a common point, forming a pyramidal frustum (see *Geometric Demonstration* section). Therefore, all the virtual axes can form a network with a shared point to each vertex, making it easier to compute their positions from the target surface mesh. Figure 11 demonstrates the basic computational workflow for our algorithm which we will explain in detail through the following three parts (Algorithm A in the Appendix).

Computation of Virtual Rotation Axes

The aim of the first part of our algorithm is to compute the location of all the virtual rotation axes during the reconfiguration. To ensure all the polygons stay in the same plane after unrolling, the network of virtual axes must follow the edge directions in the 'neutral surface' to the original conical mesh, as demonstrated by Chiang (2019). All the normal vectors of the mesh face on the 'neutral surface' have half as many polar angles as their corresponding

faces in a polar coordinate system whose z-axis points towards the common plane after face unrolling. Here we provide an algorithm to compute the neutral surface, which expands and augments the original research (Algorithms B and C in the Appendix).

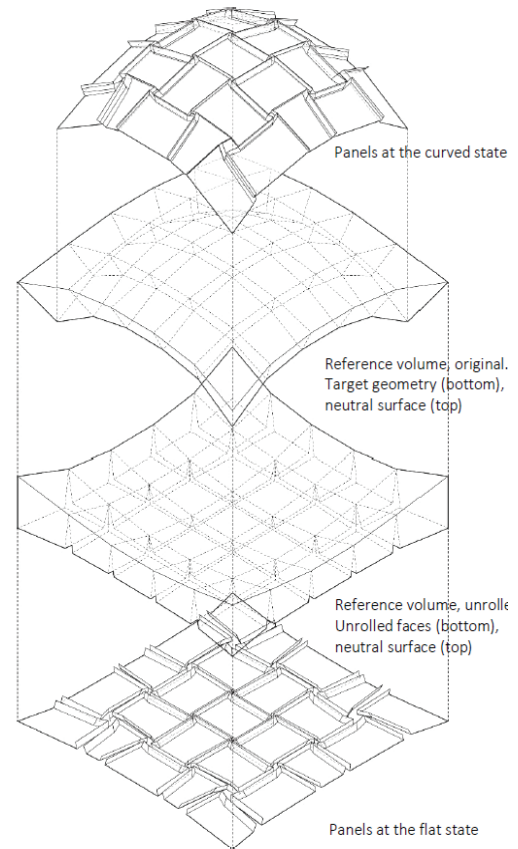
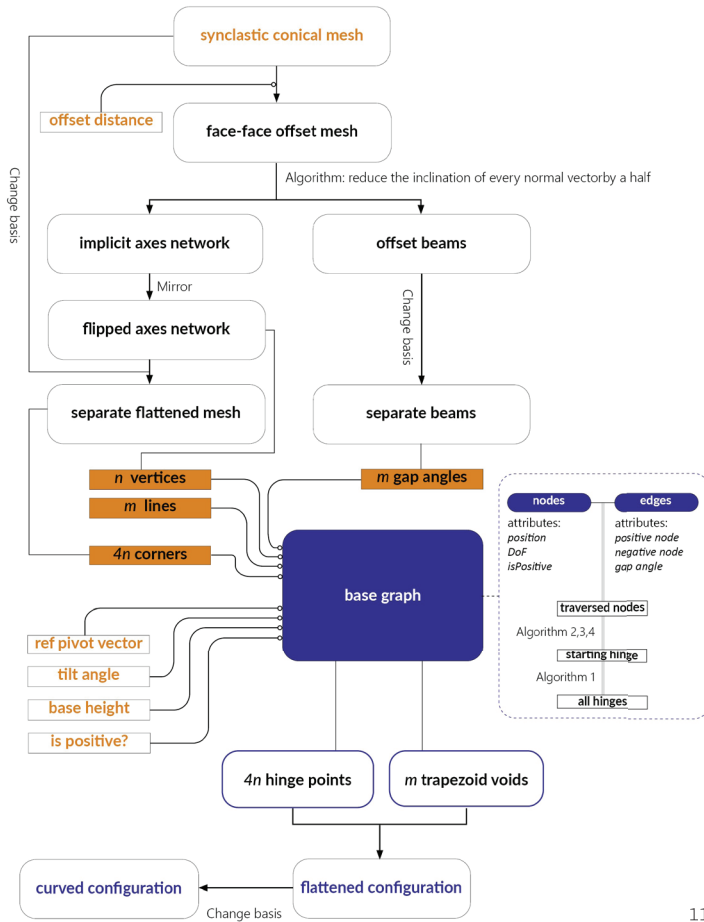
After computing the neutral surface, we can mirror it against the plane defined by the z-axis of the polar coordinate system to get its reference position after reconfiguration. Since the relative positional relationship stays unchanged in the reconfiguration, we can then unroll each polygon from the target surface to separate planar positions by orienting through its virtual rotational axes.

Computation of Skewed Hinges

The second part of our algorithm is to locate the hinges in the rotating connectors. The hinges will transform the unrolled panels that are computed in the first phase. The hexagonal voids created by those rotators bring all scattered panels in the flat configuration back to the initial curved configuration.

Figure 13 shows how we locate the aligned hinge positions at a vertex through an internal loop. The loop cycles depend on the vertex valence. Each hinge should also pass through the merged point on the neutral plane. All the hinges should also be limited in the bisector plane to the reference rotation angles centered at the merging point to glue each unrolled vertex together. Recall the angle constraints in Equation (3). Since θ_n is equivalent to the dihedral angle of the associated edge at curved stage due to the desired edge transformation, we can get ω_{n-1} for each hexagonal void. With each hinge fixed to the bisector plane, once we know the location of one hinge, we can compute the adjacent hinge by finding a direction from its constrained plane so that this hinge connects the calculated hinge and its associated edge at the 'neutral surface' to a dihedral angle ω_n . Algorithm D in the appendix shows how we compute it through linear algebra. As we loop through to the last hinge, the same operation should bring that hinge to the exact position at the starting point.

However, this looping algorithm may need different input variables to execute, subject to the DoFs limited by the vertex's neighborhood condition. The initial input without external constraints has three DoFs: two for pivot vector direction and one for angular coordinates at the bisector plane. Each time we fix a vertex's rotation connector, we eliminate one DoF of its neighborhood. Figure 14 explains the different cases of this input with DoFs at 3,2,1, respectively.



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To assume a vertex will never have a zero-input variable, we apply a spanning tree to consecutively fix the hinge positions at each vertex across the mesh. The propagation follows a very straightforward approach by placing neighborhood vertices in the candidate bag and removing executed vertex from the pool (Algorithm E in the Appendix). It is noticed that the boundary vertices come at the very last step of this enumeration because they have fewer limits to constrain the hinge directions.

Computation of Final Configuration States

The final part of our algorithm is to form the patterned panels by connecting the designed hinges. We can design two patterns from the same set of skewed hinges based on whether the flat stage is open or closed, as shown in Figure 15. If we want to actuate the system by compressed stress, the flat pattern should be open with hexagonal voids. Here we can mirror each edge of the rotating connectors to evaluate the resting positions for those hinges after the bi-stable transformation phase and connect adjacent hinges and resting positions along corresponding edges to form those hexagonal voids and thus all the panel geometries. In the other situation where an open curved configuration is desired, we connect the hinge positions

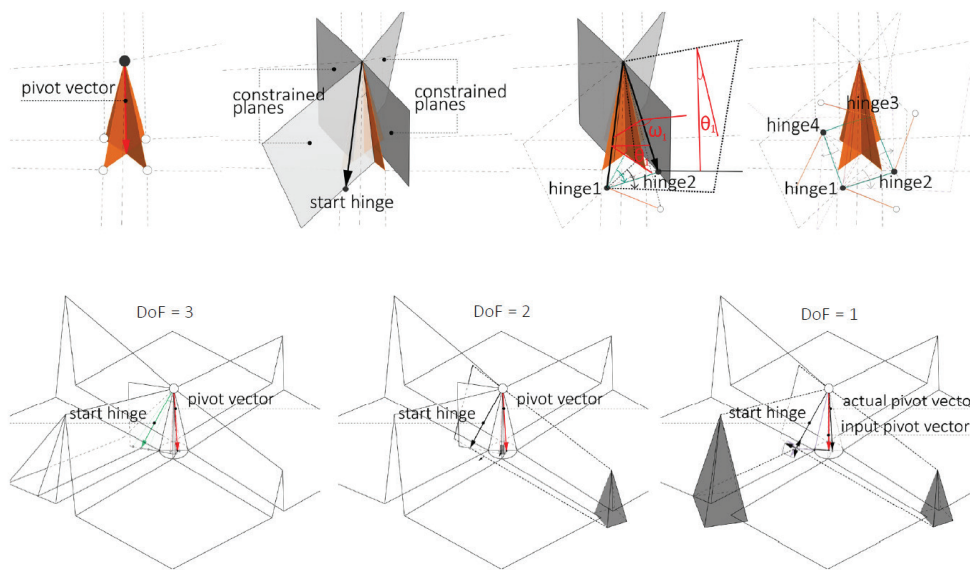
in an orthogonal way and leave no gap in the flat pattern. After we get those panels, we use the transformation introduced in *Geometric Demonstration* section to move the patterns from the plane to the curved configuration.

This section presents the computational methods to design bi-stable auxetic patterns for a target synclastic polyhedral surface. We provide a pivotal diagram showing the relationship between the reference polyhedral geometry, virtual axes, and our designed results (Figure 12).

RESULTS

Evaluation and Optimization

The proposed workflow has been tested on several architectural surfaces as shown in Figure 2. Important criteria to evaluate the performance of our workflow are the offset distance at each panel edge from its original edge and the acute angle at each hexagonal void. Those two values are subject to the influence of both intrinsic geometric properties and the input variables in our algorithm. We introduce a non-linear optimization strategy, namely through the guided projection method (Tang 2014) to adjust those variables to better design the patterns.



- 11 Computational workflow of bi-stable auxetic design
- 12 Correspondence between target and reference geometry and the bi-stable auxetic patterns
- 13 Looping algorithm to align hinge positions inside a rotating connector. (Left to right) (1) Determine isosceles triangles by pivot direction; (2) draw bisector planes of the tip angle, determine direction of the starting hinge; (3) get next hinge direction according to the constraint; and (4) loop (n-1) times (n = vertex valence) to get all the hinge positions
- 14 Different DoFs in a rotating connector design

14

Fabrication Results

We used CNC milling to materialize the pattern generated by our algorithm. We paid careful attention to the drilling width as a design factor. The diameter of the drill bit leads to round corners for acute angles of those hexagons, resulting in unwanted gaps between voids. Hence, we offset the edges so that the drill can create usable living hinges (Figure 16). Parameters are scripted and tested before finding the proper hinge width values at 0.3mm and offset distance at 1.0 mm to fabricate 1/8in-thick polypropylene sheet. The final prototype model fabricated by a 6-axis CNC machine is shown in Figure 17.

DISCUSSION

Structural Behavior

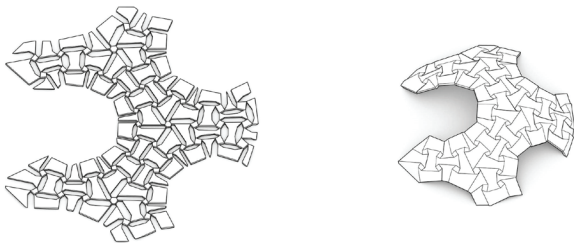
During the process of reconfiguration, all the hinges will endure high centralized stress and strain. However when the second stable state is reached, all the panels spring back to their resting length. Particularly, when the curved state is in contraction (Figure 15a), beams of each panel will touch on their neighborhood due to the well-designed mirrored features in our algorithm. Therefore, all of its self-weight can pass through those contact surfaces. If the target geometry is a compression-only shell, this system is capable of large gravitational loads, such as freshly poured concrete. This feature makes our system viable to serve as potential flexible, lightweight, economical and deployable formworks.

Architectural Speculation

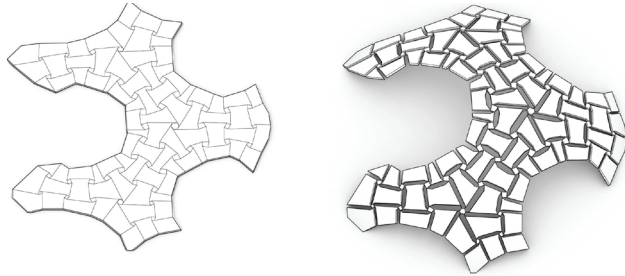
Our proposed system can be applied in various architectural conditions. Importantly, it can serve as a deployable

formwork, as it fits the structural behavior we just discussed. This type of system has been of recent interest in research, including knit-cable systems (Popescu et al. 2018) and inflatable systems (Panetta et al. 2021). Compared to the knit-cable and fabric techniques, which can only serve for minimal surfaces, our system provides solutions for compressive structures with positive-Gaussian curvature. Uniquely to those approaches, our system provides a network of repetitive geometric motifs. The interlocking pattern functions mechanically as a deployable system and provides its unique aesthetic qualities. Figure 18 shows the workflow of building a concrete shell structure through our formwork system. The volumes with gaps can be prefabricated in the factory as an injection mold, into which an elastic material like silicon rubber can be poured to fabricate the large-scale molding cast. On the construction site, a wheel-track system can provide actuation. The mold in flattened configuration becomes a curved vault after its anchor points are dragged inward. Mortar and structural concrete are then poured on the top of it, leaving the Guastavino-like interlocking texture in the bottom. This elastic formwork can be removed after giving cuts alongside the four mirror axes and flattened individually. Thus, the molding can be recycled for reproduction.

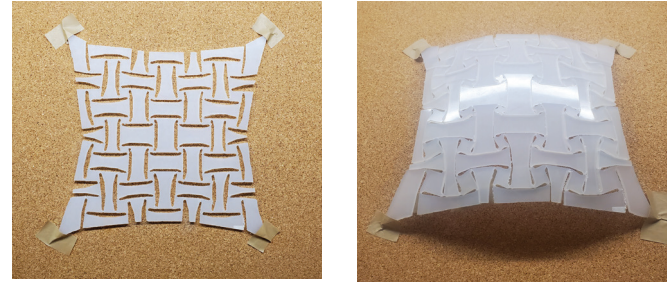
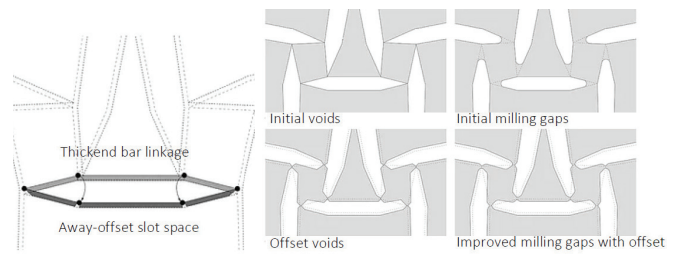
Another interesting application is to build more efficient responsive screens. Our adaptive system allows for the production of unit components that can be switched from flat to various curved shapes. More importantly, with the bi-stable mechanism in our system, energy is only needed during the transformation phrases. This is a significant advantage over the lengthy actuation process for most



(a)



(b)



existing kinetic façade or rooftop systems. For more general purposes, our system also serve as an alternative option for emergency or disaster relief shelters and as demonstrators in pavilion design (Figure 19).

Limitations and Future Work

Our research work gives primary focus on to geometrical features at start and end states, assumes all the hinges rotate freely during the reconfiguration process. The system needs extra design on hinges to resolve the stress concentration for large-scale applications. In the future, it may be helpful to add simulation and FEA analysis focusing on the hinge detailing, dynamic effects during the transformation, and structural performance in the deployed stated.

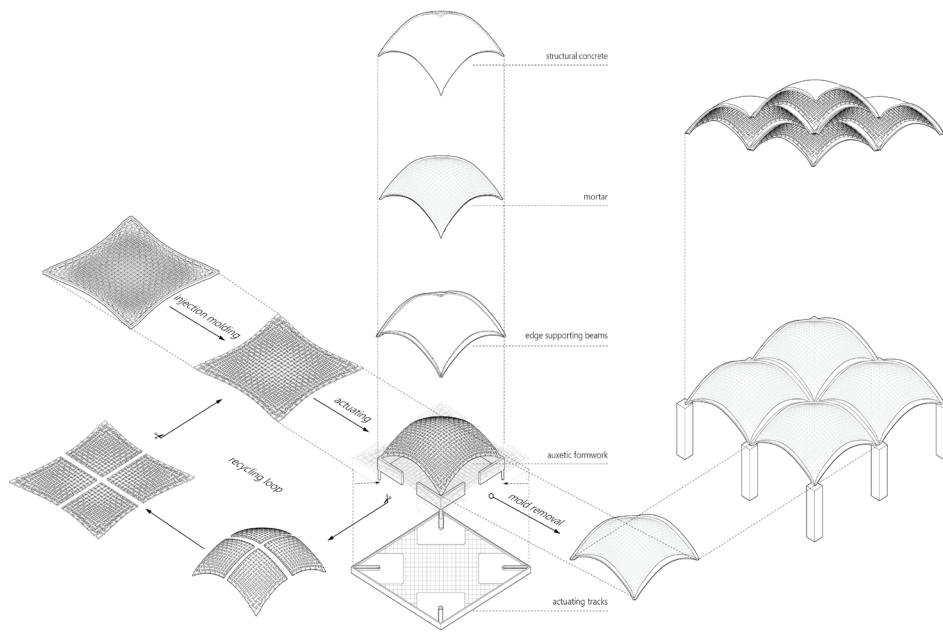
Although we demonstrated that the vertex transformation can cover both positive and negative Gaussian curvature, the nature of a 'neutral surface' limits our reference virtual hinges to the one side of our target geometry. Thus, our algorithm can only apply to synclastic geometry, It would be useful to generalize the condition of 'neutral surface' to a broader context as a network of curves with special curvature relationships to the target mesh so that we can automate the bi-stable auxetic patterns for all free-from geometries.

Currently all the edges in our pattern are bi-stable. This means a significant amount of external forces are needed to actuate the whole system, which may be inappropriate especially when we scale it up to an architectural context. For that application, we could instead mix bi-stable edge

patterns with other passive elements. This could help increase the scaling ranges and application possibilities.

CONCLUSION

We introduce geometric properties and a method for the design of bi-stable auxetic patterns. Such patterns are formed by linear hinges allowing out-of-plane rotation and thus enable the system to be actuated into shape with desired dihedral angles at each edge. Thanks to a proper propagation approach, we can generate the patterns and overcome the complex geometric condition around each vertex star. We then provide a tool for designing free-form synclastic geometry and use prototypes to showcase different fabrication and material. Our proposed system has potential to be widely used for architectural applications as a novel strategy for bi-stable deployable structures.



18



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- 15 Stable states of actuation: (a) by compression, (b) by expansion
- 16 Post-processing the geometric pattern for CNC milling
- 17 Photography of prototype model by 3mm polypropylene
- 18 Flowchart of making a flexible molding formwork
- 19 Rendering of a potential shell pavilion design

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IMAGE CREDITS

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Figure 6: Adapted with permission from Ahmad Rafsanjani.

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